Class-X

Mathematics Standard (041)

2

Section-A

Ques1 To prove:- ORPQ is a square i.e $\angle O=\angle Q=\angle P=\angle R=90^\circ$
and $OQ=QP=PR=OR$

Proof:- $\angle OQP=90^\circ$] Tangent is perpendicular to — ①
$\angle ORP=90^\circ$] the radius at the point of contact.

Now, In $\triangle ORP$ and $\triangle OQP$
$OP=OP$ (common)
$OR=OQ$ (Radii of same circle)
$PR=PQ$ (Tangents from an external point to a circle are equal)
Therefore, $\triangle ORP \cong \triangle OQP$ by SSS rule
$\therefore \angle OPR=\angle$ ~~[illegible]~~ OPQ (By CPCT)

$\Rightarrow \angle OPR=\angle OPQ=45^\circ$
$\Rightarrow \angle OPR+\angle OPQ=90^\circ$
$\Rightarrow \angle QPR=90^\circ$ — ②

3

In quad OQPR

$\angle OQP + \angle QPR + \angle PRO + \angle ROQ = 360^\circ$ (Angle Sum Property)

$90 + 90 + 90 + \angle ROQ = 360^\circ$

$\angle ROQ = 360 - 270$

$\angle ROQ = 90^\circ$ — ③

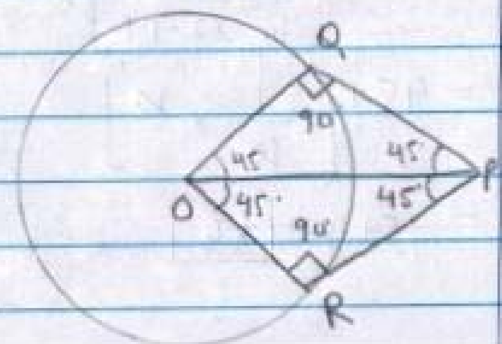
Also $OR = PR$ — ④

(As $\angle ROP = \angle RPO = 45$) (Isosceles triangle property)

Also $QP = PR$ (Tangents from an external point to a circle are equal) — ⑤

⇒ Using ④ and ⑤

$OQ = PQ = PR = OR$ — ⑥



Using ①, ②, ③, ⑥

ORPQ is a square.

Hence proved.

180
$\underline{-90}$
$\underline{\ 90}$

Ques 2

Class		Frequency
15-25		6
25-35		11
35-45	f_0	22
45-55	f_1	23 → Modal class
55-65	f_2	14
65-75		5

$$\text{Mode} = l + \left[\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right] \times h$$

$$= 45 + \left[\frac{23 - 22}{46 - 22 - 14}\right] \times 10$$

$$= 45 + \left(\frac{1}{\cancel{10}} \times \cancel{10}\right)$$

$$= 45 + 1 = \boxed{46}$$

$$\begin{array}{r} 23 \\ \times 2 \\ \hline 46 \\ -22 \\ \hline 24 \\ -14 \\ \hline 10 \end{array}$$

$$\begin{array}{r} 46 \\ -22 \\ \hline 24 \\ -14 \\ \hline 10 \end{array}$$

Ques 3

$a = 10$

$S_{14} = \frac{n}{2}\left(2a + (n-1)d\right)$

$1505 = \frac{14}{2}\left(2(10) + 13d\right)$

$1505 = 7(20 + 13d)$

$\frac{1505}{7} = 20 + 13d$

$215 = 20 + 13d$

$\frac{195}{13} = d$

$\underline{\underline{d = 15}}$

$7\overline{)1505}\ 215$
-14
10
-7
35

215
-20
195

$13 \times 5 = 65$

$13\overline{)195}\ 15$
-13
65

Ques 4

$9, 7, 5$ —— and $15, 12, 9$ ——

$a_n = a + (n-1)d$
$= 9 + (n-1)-2$
$= 9 - 2n + 2$
$= 11 - 2n$

$a'_n = a' + (n-1)d'$
$= 15 + (n-1)-3$
$= 15 - 3n + 3$
$= 18 - 3n$

Since $a_n = a'_n$

$11-2n = 18-3n$

$11-18 = -3n+2n$

$-7 = -n$

$\boxed{n=7}$

Ques 5

(a) $x^2 - 2ax - (4b^2 - a^2) = 0$

$b^2 - 4ac = 4a^2 - 4[-(4b^2 - a^2)](1)$

$= 4a^2 - 4[-4b^2 + a^2]$

$= \cancel{4a^2} + 16b^2 \cancel{- 4a^2}$

$= 16b^2$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{2a \pm \sqrt{16b^2}}{2(1)} = \frac{2a \pm 4b}{2}$

$\Rightarrow x = \frac{2a+4b}{2}$ $x = \frac{2a-4b}{2}$

$x = a+2b$ $x = a-2b$

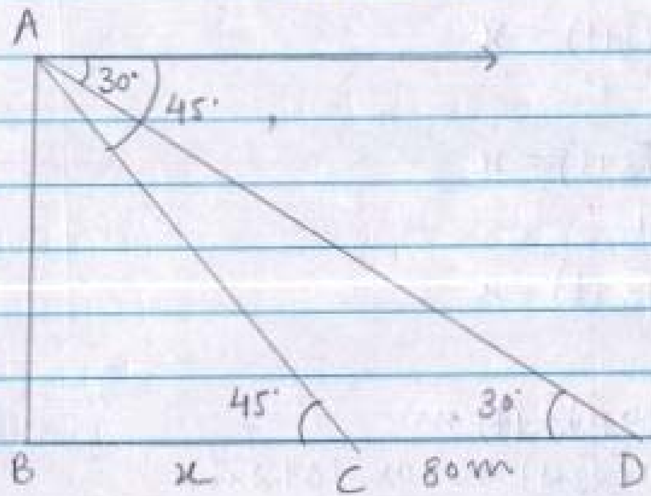
Ques 6

(b) Length of cuboid = 18 cm
Breadth of cuboid = 6 cm
Height of cuboid = 6 cm

TSA of the cuboid $= 2(lb + bh + hl)$
$= 2(18 \times 6 + 6 \times 6 + 6 \times 18)$
$= 2(108 + 36 + 108)$
$= 2(252)$
$= \underline{504\ cm^2}$

Section-B

Ques 7



$18 \times 6 = 108$
$108 + 108 = 216$
$216 + 36 = 252$
$252 \times 2 = 504$

8

In ΔABC

$\tan 45 = \frac{P}{B} = \frac{AB}{x}$

$1 = \frac{AB}{x}$

$\boxed{AB = x}$

In ΔABD

$\tan 30 = \frac{P}{B} = \frac{AB}{80+x} \Rightarrow \frac{1}{\sqrt{3}} = \frac{AB}{80+x}$

$\frac{80+x}{\sqrt{3}} = AB$

$\frac{80+x}{\sqrt{3}} = x$

$80+x = \sqrt{3}x$

$80 = \sqrt{3}x - x$

$80 = x(\sqrt{3}-1)$

$\frac{80}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1} = x$

$\frac{80(\sqrt{3}+1)}{(\sqrt{3})^2 - 1^2} = x$

$\frac{80(\sqrt{3}+1)}{3-1} = x$

$\frac{80(\sqrt{3}+1)}{2} = x$

$x = 40(\sqrt{3}+1)$ m

$\Rightarrow AB = 40(\sqrt{3}+1)$ m or 109.2 m

1.73
+1
2.73

2.73
×4
1092

1.73
+1
2.73

2.73
×40
000
1092×
1092.0

273
×40
000
1092×
109.20

Ques 8

Class	Frequency	C.F.
1400–1550	6	6
1550–1700	13	19
1700–1850	25	44 → Median class
1850–2000	10	54 = n

$$\text{Median} = l + \left(\frac{\frac{n}{2} - CF}{f}\right) \times h$$

$$= 1700 + \left(\frac{27-19}{25}\right) \times 150$$

$$= 1700 + \left(\frac{8}{\cancel{25}_{5}}\right) \times \cancel{150}^{\cancel{30}\ 6}$$

$$= 1700 + 48$$

$$= \underline{\underline{1748}}$$

$13 + 6 = 19$

$19 + 25 = 44$

$\frac{54}{2} = 27$

$1850 - 1700 = 150$

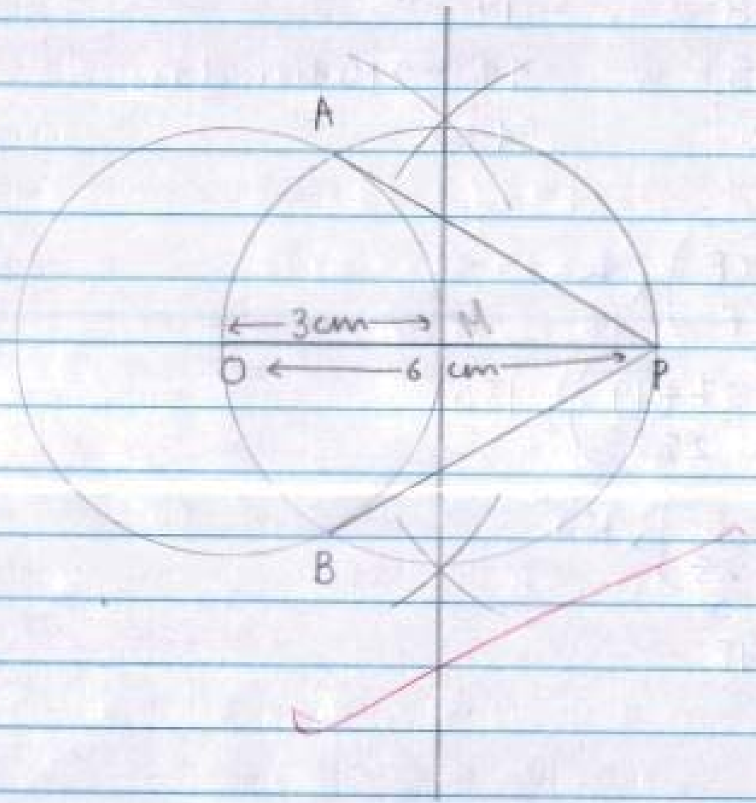
$27 - 19 = 8$

$19 + 25 = 44$ $\quad 1850 - 1700 = 150$

$\frac{54}{2} = 27$

$27 - 19 = 8$

Ques 9
(b)



Steps of construction →

1. Draw a circle with O as centre and radius 3cm.
2. Take a point P outside the circle at a distance of 6cm from its centre O.
3. Join OP.
4. Construct the perpendicular bisector of OP.
5. Name the mid-point of OP as M.
6. Draw a circle with M as centre and radius OM.
7. The second circle intersects the first circle at two points. Name them A and B
8. Connect PA and PB.
9. PA and PB are the required tangents.

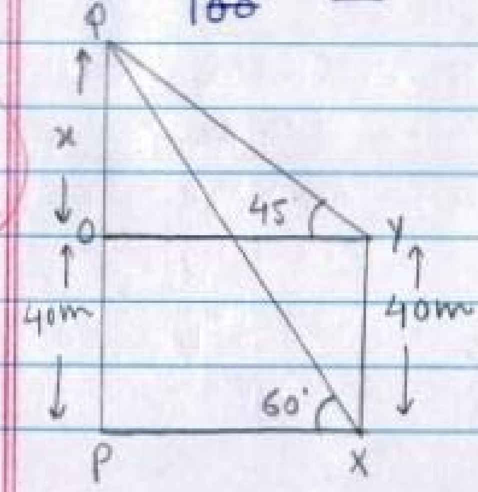
Ques 10

Class	Frequency (f_i)	x_i	$f_i x_i$
0-10	12	5	60
10-20	18	15	270
20-30	27	25	675
30-40	20	35	700
40-50	17	45	765
50-60	6	55	330
	100		2800

$$\text{Mean} = \frac{\Sigma f_i x_i}{\Sigma f_i}$$

$$\bar{x} = \frac{28\cancel{00}}{1\cancel{00}} = \underline{\underline{28}}$$

Ques 11



In ΔQOY

$$\tan 45 = \frac{P}{B} = \frac{QO}{OY} = \frac{x}{OY}$$

$$1 = \frac{x}{OY}$$

$$\boxed{OY = x} \Rightarrow \boxed{PX = x}$$

$\frac{130}{2} = 65$

$\frac{50}{2} = 25$

$\frac{110}{2} = 55$

$18 \times 15 = 270$

$25 \times 27 = 675$

$35 \times 20 = 700$

$45 \times 17 = 765$

$55 \times 6 = 330$

In ΔQPX

$$\tan 60 = \frac{P}{B} = \frac{QP}{PX} = \frac{40+x}{x}$$

$$\sqrt{3} = \frac{40+x}{x}$$

$$\sqrt{3}x = 40+x$$

$$\sqrt{3}x - x = 40$$

$$x(\sqrt{3}-1) = 40$$

$$x = \frac{40}{(\sqrt{3}-1)} \times \frac{\sqrt{3}+1}{\sqrt{3}+1} = \frac{40(\sqrt{3}+1)}{(\sqrt{3})^2-1^2} = \frac{40(\sqrt{3}+1)}{3-1}$$

$$x = \frac{40(\sqrt{3}+1)}{2}$$

$$x = 20(\sqrt{3}+1)\,m$$

$$x = 54.6m$$

$$\Rightarrow \boxed{PX = 54.6m}$$

$$PQ = 40+x$$

$$= 40+54.6$$

$$= \boxed{94.6m}$$

$1.73 + 1 = 2.73$

$273 \times 2 = 5460$

273×20: 000, $546\times$, 5460

$54.6 + 40 = 94.6$

18×15: 90, $18\times$, 270

$1.73 + 1 = 2.73$

27×25: 135, $54\times$, 675

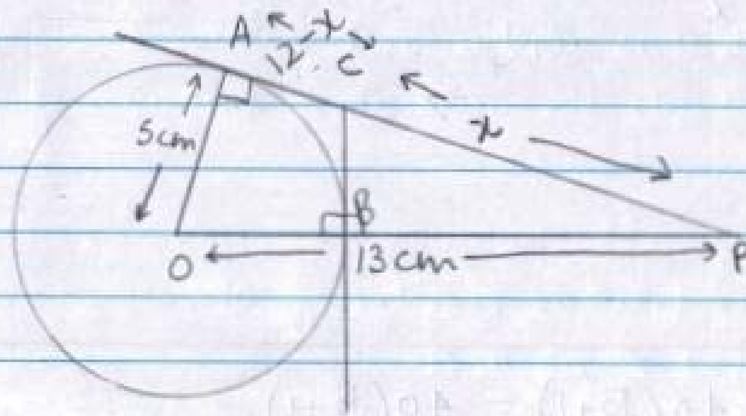
45×17: 315, $45\times$, 765

$35 \times 2 = 70$

$55 \times 2 = 110$

Ques 12

(b)



$\angle OAP = 90°$ (Tangent is perpendicular to radius at the point of contact)

Using pythagoras theorem in ΔOAP

$H^2 = B^2 + P^2$

$OP^2 = AP^2 + OA^2$

$13^2 = AP^2 + 5^2$

$169 = AP^2 + 25$

$169 - 25 = AP^2$

$\Rightarrow AP^2 = 144$

$\boxed{AP = \sqrt{144} = 12cm}$

Let PC be x and AC be $12-x$

$AC = BC = 12 - x$ (Tangents from point C to the circle are equal)

Also $OP = 13\,cm$

$OB = 5\,cm$

$BP = OP - OB$

$BP = 8\,cm$

$\angle OBC = 90°$ (Tangent is perpendicular to the radius at the POC)

$\Rightarrow \angle CBP = 90°$ (Linear Pair)

Using pythagoras theorem in Δ CBP

$H^2 = B^2 + P^2$

$CP^2 = BP^2 + BC^2$

$x^2 = 8^2 + (12-x)^2$

$x^2 = 64 + 144 + x^2 - 24x$

$24x = 64 + 144$

$24x = 208$

$x = \cancel{\frac{208}{24}}\ \cancel{\frac{104}{12}}\ \frac{52}{6}\ \frac{26}{3}$

$BC = 12 - x$

$BC = 12 - \frac{26}{3}$

$BC = \frac{36-26}{3} = \frac{10}{3} = \boxed{3.34\,m}$

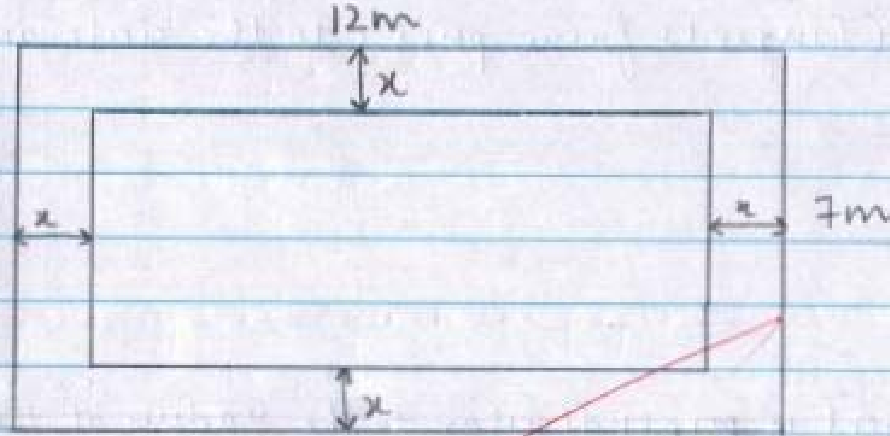
144
$+64$
208

144
64
208

$\frac{208}{24}$ $\frac{26}{\cancel{52}}$ $\frac{\cancel{104}}{\cancel{12}}$ $\cancel{6}$ 3

$3 \overline{)10}$ 3.3
-9
10

Ques 13



~~Length of pool = (12-2x) m~~
~~Breadth of pool = (7-2 x) m~~

Dimensions of pool — $(12-2x)$ and $(7-2x)$

(a) Area = Length × Breadth

$36 = (12-2x)(7-2x)$

$36 = (84 - 24x - 14x + 4x^2)$

$36 = 84 - 38x + 4x^2$

$36 - 84 = -38x + 4x^2$

$-48 = -38x + 4x^2$

$4x^2 - 38x + 48 = 0 \Rightarrow 2x^2 - 19x + 24 = 0$

$\begin{array}{r} 19 \\ \times 2 \\ \hline 38 \end{array}$ $\begin{array}{r} 12 \\ \times 7 \\ \hline 84 \end{array}$ $\begin{array}{r} 24 \\ +14 \\ \hline 38 \end{array}$ $\begin{array}{r} 84 \\ -36 \\ \hline 48 \end{array}$ $\begin{array}{r} 84 \\ -36 \\ \hline 48 \end{array}$ $\begin{array}{r} 84 \\ -36 \\ \hline 48 \end{array}$

(b) $2x^2-19x+24=0$

$2x^2-16x-3x+24=0$

$2x(x-8)-3(x-8)=0$

$x=\frac{3}{2}$ $x=8$

$x=8$ will be neglected because if $x=8$ the ~~length of~~ width of sidewalk ~~edge~~ will become more than 16 but they are already given 7m and 12m the dimensions

$\therefore$ $\boxed{x=1.5m}$

48
1×48
2×24
3×16

24
×24
96
48×
576
+49
625

Ques14

(a) CSA of one cap = Square cm paper required for one cap

$2\pi r=44$

$2\times\frac{22}{7}\times r=44$

$r=\frac{44\times7}{22\times2}=7cm$

$l^2=h^2+r^2$

$l^2=24^2+7^2$

$l^2=576+49$

$l^2=625$

$l=25cm$

CSA of one cap $= \pi r l$

$= \frac{22}{\cancel{7}} \times \cancel{7} \times 25$

$= 550\ cm^2$

CSA of four caps i.e. square cm paper required for four caps $= 4 \times 550$

$= 2200\ cm^2$

(b) Volume of cake $= \pi r^2 h$

$= \frac{22}{\cancel{7}} \times 12 \times 12 \times 14\ 2$

$d = 24cm$

$r = \frac{d}{2} = 12cm$

$= 6336\ cm^3$

$650\ cm^3 = 100g$ or $0.1kg$

$6336\ cm^3 = \frac{0.1\ kg \times 6336}{650} = \frac{6336}{6500} = \frac{95.9}{100} = 0.95\ kg$

So they should order a 1 kg cake.

$25 \times 22 = 50 + 500 = 550$

$\frac{550}{25} = 22$

$55 \times 4 = 2200$

$65 \times 5 = 325$

$65 \times 8 = 520$

$144 \times 44 = 576 + 5760 = 6336$

$65 \times 9 = 585$

$144 \times 44 = 576 + 5760 = 6336$

$13 \times 4 = 52$

$13 \times 8 = 104$

$12 \overline{)6336}$ 48, -52, 113, -104, 96